Rapid single flux quantum logic using π-shifters

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We have found that the size of some Rapid Single Flux Quantum (RSFQ) logic cells based on conventional 0-type Josephson junctions can be significantly reduced by using π-type junction as a phase shifter in *passive* (non-switching) *mode*. In comparison with the recently suggested active (switching) π-junctions mode , the passive mode offers much greater operation margins for their critical current I_c^π. This gives π-junctions a chance to be implemented in RSFQ designs in the near future. As an example, we have simulated the operation of a toggle flip-flop with *zero geometrical inductance* of the fluxon storage loop. Simulations show that the parametric inductance of the π-junction and its normal resistance R_n form a low-pass filter, which sets the low limit for π-junctions $I_c^\pi R_n$ product, but offers wide range variation of the other parameters. Possible reduction of RSFQ cell size by using π-junctions opens the way to scale superconducting logic circuits down to the sub-micron dimensions.

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At variance to standard Josephson junctions, π-junctions are biased in their ground state by a π phase shift of the superconducting order parameter. This phase shift can be established by using various types of junction fabrication techniques which have been recently demonstrated. The π-junctions can be manufactured using either high-T_c [1] or low-T_c superconductors [2–4], or using a combination of both materials [5]. The junctions combining high-T_c superconductors are based on unconventional order parameter symmetry, which is common for most of these materials. The most robust low-T_c realization of the π-junctions is based on Cooper pairs tunneling through magnetic states, which induce spatial oscillations of the order parameter in the barrier.

The first approach to use π junctions in superconducting digital circuits has been proposed by Terzioglu and Beasley [6]. The key point in their concept is similar to semiconducting CMOS logic. It is based on using Superconducting Quantum Interference Devices (SQUIDs) with π-junctions, whose transport properties are complementary to conventional SQUID behavior. An elegant way to circumvent flux biasing needed for complimentary logic was achieved by introducing a π-junction in the SQUID loop. Complimentary superconducting logic is not widely developed due to its basic speed limitation. The idea of using π-junctions in RSFQ logic to reduce inductances of some circuits was suggested by one of the authors [7, 8], and later was further developed at the University of Twente [9]. However, the most straightforward approach of using π-junctions as *active switches* [7–9] requires a combination of conventional junctions (0-junctions) with π-junctions, and their parameters such as their critical current I_c, and normal state resistance R_n have to be similar, and have to be within the same operating margins. It is rather difficult to satisfy these stringent requirements by using currently available π-junction fabrication technology.

To overcome this problem, we propose to use π-junctions as *passive phase-shifters* in RSFQ circuits. The major advantage of this approach is that only conventional junctions are used to carry flux quanta, and to provide logical functionality. The relatively large geometrical inductance of RSFQ cells - which is required by single flux quantum (fluxon) storage - can be substituted by the π-junctions operated in the passive regime. In this case they remain in the superconducting state and do not switch during circuit operation. This mode has an advantage over the active mode [7–9] in that the operating margins of the π-junctions critical currents become extremely wide. The critical current I_c^π has to be larger than the critical current I_c^0 of conventional 0-type junctions, and the $I_c^\pi R_n$ product must be larger than one half of the corresponding value of the conventional junctions. Since the critical current densities j_c for 0-junctions and π-junctions do not need to be the same, one can relatively easily combine different junction fabrication technologies.

The standard part of any flux-memory RSFQ cell contains at least two Josephson junctions connected by an inductance L, as illustrated in Fig. 1(a). This circuit has two distinct states - one without a fluxon (no circulating current in the SQUID loop), and the other with a fluxon, which produces circulating current within the loop [10]. An operating constraint for RSFQ cells is $I_cL/\Phi_0 > 1$, where $\Phi_0 = 2.07 \cdot 10^{-15}\,\mathrm{V\cdot s}$ is the magnetic flux quantum. It simply means that the circulating current due to stored flux must be smaller than the critical currents of the junctions. For the most common technology with the critical current density $j_c = 1\,\mathrm{kA/cm^2}$ [11] it translates into cell inductance L of about $10\,\mathrm{pH}$, and its respective size of hundred micrometers. This basic constraint does

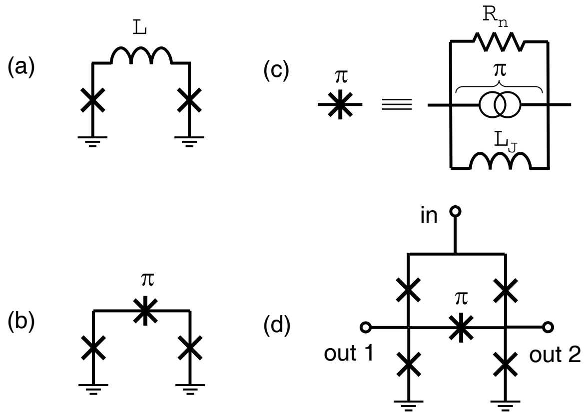

FIG. 1: (a) Two-junction interferometer (SQUID) with loop inductance $L$is a part of fluxon memorizing RSFQ cells. (b) Same SQUID with inductance replaced by a π-shifter. (c) Equivalent circuit of π-shifter. (d) Schematic view of a Toggle Flip-Flop (T-F-F) with zero loop inductance.

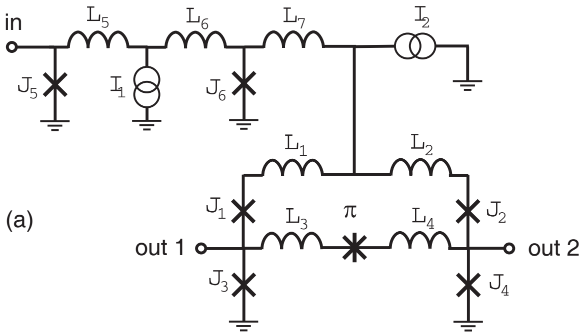

FIG. 2: The simulated circuit containing T-F-F with a π-shifter. Circuit parameters are based on Hypres 1 kA/cm^2 [11]fabrication technology with $I_cR_n = 0.295\,\text{mV}$. $L_1 = L_2 = L_3 = L_4 = 0.26\,\text{pH}$, $L_5 = L_6 = 1.3\,\text{pH}$, $L_7 = 1.56\,\text{pH}$, $I_1 = 0.24\,\text{mA}$, $I_2 = 0.1\,\text{mA}$, $J_1 = 0.2\,\text{mA}$, $J_2 = 0.166\,\text{mA}$, $J_3 = 0.154\,\text{mA}$, $J_4 = 0.219\,\text{mA}$, $J_5 = 0.213\,\text{mA}$, $J_6 = 0.406\,\text{mA}$. Critical current of the π-junction is $I_c^\pi = -0.3\,\text{mA}$ and $I_c^\pi R_n = -0.05\,\text{mV}$

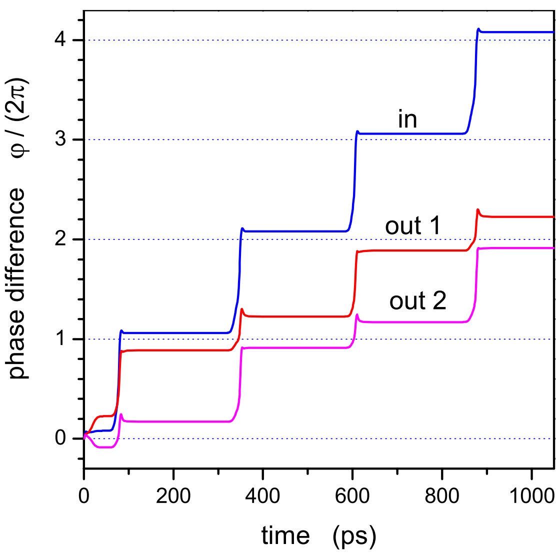

FIG. 3: Numerical simulation of the circuit shown in Fig. 2. The sequence of output switches (vertical 2π leaps) demonstrates successful operation of the T-F-F.

not permit reducing the size of RSFQ cells.

We propose to replace the inductance L in Fig. 1(a) by a π-junction as shown in Fig. 1(b). A circuit which is approximately equivalent to the π-junction (operated in the passive mode as a phase shifter) is presented in Fig. 1(c). It consists of a virtual current source which ultimately biases the Josephson inductance $L_J = \Phi_0/(2\pi I_c^\pi \cos\varphi)$ such that the phase shift φ on the isolated junction is maintained to be π. Formally, this condition corresponds to the negative critical current [8] of the junctions because $I_c = I_c^\pi \cos\pi = -I_c^\pi$. In the circuit as in Fig. 1(b), π-junction creates a circulating current, whose direction can be set, for example, by biasing one of the junctions with a small bias current. If the critical currents of all three junctions are the same, enclosing them within a loop as shown in Fig. 1(b) sets the modulus of the phase difference across each junction to be $\pi/3$ accordingly to their inductances. Thus, the entrance of a single flux quantum into the circuit simply reverses the direction of the circulating current. For storing digital information, one can now use two stable states of the cell which differ by the direction of the persistent current. It is worth mentioning that such a π-junction loop in the quantum limit forms a two-state system, which has been proposed earlier to be used as a qubit [12].

In spite of the fact that in the proposed circuit π-junction does not switch to the resistive state, its normal shunt resistance R_n and Josephson inductance L_J shown in Fig. 1(c) form a low-pass filter with time constant $\tau_\pi = L_J/R_n = \Phi_0/|2\pi I_c^\pi R_n \cos\varphi|$. This affects the ac currents in the π-junction which occur due to the switching of the other two junctions during operation, and thus sets a rigid lower limit for the π-junction's $I_c^\pi R_n$ product.

To demonstrate the operation of a circuit based on the proposed architecture, we have simulated an RSFQ Toggle Flip-Flop (T-F-F) shown in Fig. 1(d). It consists of two memory cells mentioned above, which share the same π-junction (in conventional circuit this will be an inductor). The circuit is operated in such a way that top and bottom cells are always in different states (have circulating currents in opposite directions). An RSFQ pulse (fluxon) arriving to the circuit input switches the states of both cells simultaneously, thus performing the T-F-F operation. Every second fluxon proceeds to the *output 1* and every other fluxon goes to the *output 2*.

Figure 2 shows a realistic rather than schematic T-F-F circuit, which was simulated for experimentally feasible parameters using the superconducting circuit simulator WinS [8]. Figure 3 presents the simulated phase differences at the T-F-F input and outputs. Each 2π leap in Fig. 3 corresponds to the fluxon arrival at the corresponding T-F-F output. Small inductances $L_1 - L_4$ of 0.26 pH have been added to account for the realistic geometry of the circuit, assuming that the smallest cell size is limited by the Josephson junction dimensions given by the standard design rules [11]. The complete *elimination of the inductances $L_1 - L_4$ does not affect the circuit operation margins*, which are about ±35%. Margins are calculated at 20 GHz input data rate. Note, that operational margins for π-junction critical current I_c^π and its $I_c^\pi R_n$ product have only the lower limit. With the circuit parameters indicated in the caption of Fig. 2, the upper and lower margins for $|I_c^\pi|$ are -54% and $\geq +90\%$; for $|I_c^\pi| R_n$ they are -45%, and $\geq +90\%$, respectively. McCumber parameter β_c of π-junction can vary from zero to about 10 and, in the case it would be higher, can be reduced by an external shunt resistor.

We would like to mention the targeted parameters of the π-junctions to be used in RSFQ circuitry at 4.2 K temperature. To operate at 20 GHz clock rate their $I_c^\pi R_n$ product should be at least 0.05 mV, or approximately 0.005 mV at ten times lower clock rate. The most difficult requirement would be the critical current of 0.2 mA, which may be reduced by using lower temperatures. It means that critical current density of planar (two-dimensional) π-junctions should be close to the conventional ones. This translates to the linear critical current density of about 0.5 A/cm for film-edge type (one-dimensional) π-junctions.

It should be noted that not all of the RSFQ circuits can benefit from using π-junctions. One of the important examples is a Josephson transmission line (JTL), which is widely used in RSFQ designs to connect logic cells. Substitution of JTL inductors by π-junctions does not work in this case. To reduce the geometrical size of JTL, one may consider substituting inductors with one or more conventional 0-type Josephson junctions, similar to the Josephson ladder geometry [13]. At the positive side we should mention that an RSFQ memory based on SQUIDS as memory cells can be significantly reduced in size when using π-junctions, and thus can offer much faster data access.

In summary, replacing inductors by passive π-junctions in some RSFQ cells greatly reduces their size. Potentially, it opens the way to the miniaturization of RSFQ circuits and to scaling their dimensions down to the submicron range [14]. However, the critical current density of π-junctions has to be made high, which still seems to be a challenge for existing fabrication technologies.

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